NONLINEAR EQUIVALENT SDOF ANALYSIS OF STEEL FRAME STRUCTURES USING PARAMETERS ESTIMATED FROM RECORDED FLOOR ACCELERATION RESPONSE: AN EXPERIMENTAL CASE STUDY

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ABSTRACTS: In practice, the building displacement response is often estimated from acceleration using double integration due to the expensiveness and difficulty of measuring displacement. However, a high-pass filter is needed to eliminate the low-frequency component due to the existence of indistinguishable low-frequency noise that incurred during the double integrating process. As a result, the displacements obtained from accelerations may have large errors, especially when the structure has significant nonlinear deformation. This study aims to estimate the displacement response of steel frame structures by firstly estimating SDOF hysteretic parameters from the acceleration response data, and then by performing numerical analysis using the extracted hysteresis model. The proposed approach was verified using experimental data from shaking table test of three identical 5-story steel frame structures subjected to different seismic records. The results show that the SDOF analysis maximum displacement has higher accuracy than the equivalent SDOF maximum displacement estimated from recorded floor acceleration response data that is recently used for post-earthquake building assessment.

KEYWORDS: Structural health monitoring, steel structure, shaking table, nonlinear SDOF analysis, seismic response.

1. INTRODUCTION

After a strong earthquake, aftershocks may cause extensive damage to buildings. For this reason, post-earthquake evaluations should be conducted as quickly as possible to help experts assess building safety. Not only does a speedy and reliable post-earthquake damage evaluation help identify unsafe buildings in a timely manner to reduce injuries during future seismic events, but it also has socially and economically benefits such as reducing downtime and speeding up social recovery process. For instance, if a high-tech factory was affected by a large earthquake, a rapid and accurate assessment should be performed to ensure that technicians can reenter the factory safely to recover and relocate valuable equipment and products to reduce losses arising from operational disruption.

Structural health monitoring (SHM) is a method to speed up post-earthquake damage evaluations [1] as it provides essential information on the structural response during an earthquake. While the acceleration response of a building can be measured accurately by accelerometers, obtaining displacement response is still a major challenge for researchers and engineers. Typically, double integration of the acceleration data is performed to estimate displacement response.

Kusunoki et al [2] proposed an approach for deriving an equivalent single-degree-of-freedom (SDOF) capacity curve of a building using recorded floor acceleration response and structural floor masses. The capacity curve can then be used to assess the residual seismic capacity of existing buildings and predict the damage caused by aftershocks. A flowchart of Kusunoki et al.'s [2] method for estimating the structure's capacity curve from floor acceleration response is shown in Figure 1. First, the floor displacement response was estimated from acceleration responses by double-integrating. Then, the floor acceleration and displacement response were decomposed into different frequency bands response called ranks by using discrete wavelet transform (DWT). Afterward, an automated procedure proposed by Yeow et al[3], which considers various properties of each rank such as peak displacement, peak acceleration, and stiffness, was utilized to identify ranks that contain predominant mode response and are not affected by the low-frequency noise arising from the double-integrating process. In the next step, identified ranks were combined to derive the equivalent SDOF displacement and acceleration response of the building using Eq. (1) and (2), respectively. Finally, the structure's capacity curve, also known as the skeleton capacity curve, was derived from the equivalent SDOF hysteresis response [2].

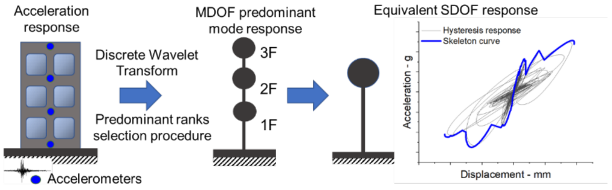


Figure 1. A flowchart of Kusunoki et al.'s method [2]

However, estimating displacement response from acceleration response using double-integrating can lead to errors due to the incurring of low-frequency noise, which may be indistinguishable from the low-frequency displacement response that represents nonlinear deformation. As a filter is required to eliminate low-frequency noise as it can cause significant overestimation of displacements, important information on nonlinear deformation may also be filtered out which could lead to an underestimation of building damage level.

$$\Delta_j^*(t) = \frac{\sum_{i=1}^{N_f}(m_i.x_{i,j}(t))}{\sum_{i=1}^{N_f} m_i} \qquad (1)$$

$$\ddot{\Delta}_j^*(t) = \frac{\sum_{i=1}^{N_f}(m_i.\ddot{x}_{i,j}(t))}{\sum_{i=1}^{N_f} m_i} \qquad (2)$$

Where, Δ_j^* and $\ddot{\Delta}_j^*$ denote the representative displacement and representative acceleration of an equivalent SDOF system, respectively; m_i is the mass of i^{th} floor; $x_{i,j}$ is the floor displacement relative to the ground of rank j; N_f is the number of floors; and $\ddot{X}_{0,j}$ is the total ground acceleration.

Pham et al. [4] proposed a procedure for correcting the capacity curve maximum displacement of steel frame structures obtained from floor acceleration data by combining it with the low-frequency displacement derived from recorded acceleration data using equivalent SDOF analysis. The method was verified using data from numerical analysis of a 5-story steel frame under different seismic records. The results show that the maximum displacement errors were reduced remarkably, especially for cases with large nonlinear deformation. However, the accuracy of the nonlinear equivalent SDOF analysis was not investigated using experimental data from shaking table tests.

This study aims to evaluate the accuracy of the equivalent SDOF analysis using experimental data from shaking table test of a 5-story steel frame subjected to three different seismic records.

2. EQUIVALENT SDOF ANALYSIS USING PHAM ET AL'S METHOD

Figure 2 shows the procedure to perform equivalent SDOF analyses using parameters estimated from acceleration data that was proposed by Pham et al. [4]. First, Kusunoki et al.'s method [2] was applied for estimating the building skeleton capacity curve from floor acceleration response. Bilinear capacity curves that best fit the skeleton capacity curve were determined by using an automatic procedure. Subsequently, an average

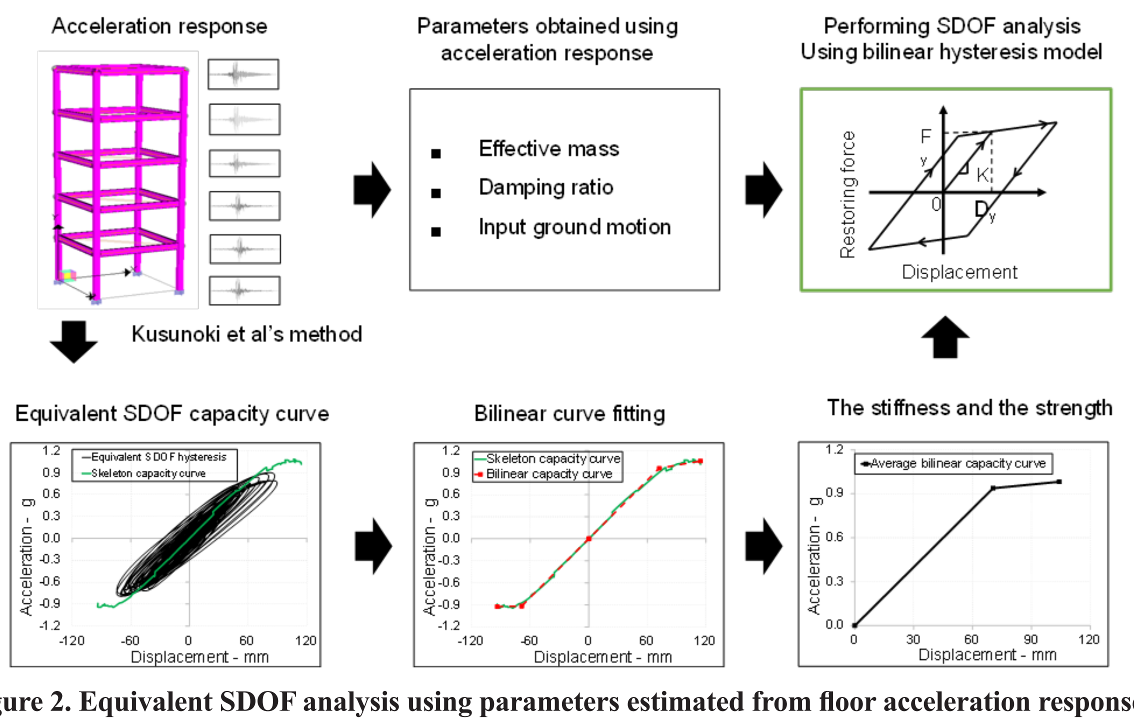


Figure 2. Equivalent SDOF analysis using parameters estimated from floor acceleration response [4]

bilinear capacity curve was calculated considering the segments in the positive and negative directions. Additionally, other parameters of the SDOF model were also estimated from the acceleration response data. The effective mass, which corresponds to maximum displacement, was estimated using Eq. 3 and was used as the mass of the SDOF model. The damping ratio was estimated using the haft power bandwidth method. The input ground motion was the recorded acceleration at the ground floor. Finally, nonlinear equivalent SDOF analysis was performed using the bilinear hysteresis model.

$$EMR(k) = \frac{\sum_{i=1}^{N_f}(m_i \times x_{i,c}(k))}{\sum_{i=1}^{N_f}(m_i \times x_{i,c}^2) \times \sum_{i=1}^{N_f} m_i} \qquad (3)$$

3. CASE STUDY DETAILS

The accuracy of the equivalent SDOF analysis was verified by using experimental data from a shaking table test of a five-story steel frame structure [5]. The steel frame was designed to form a total yielding mechanism by having yielding occur at the base of the building and at the beam ends. Its geometrical dimensions are shown in Figure 3. A photo of the specimen on the shaking table is shown in Figure 4. The beam and columns were connected to rigid steel block masses (weight of 0.15 kN) using bolted connections. The floor acceleration responses were recorded by accelerometers placed at the mid-span of each long beam. Displacement response was measured using motion tracking of targets attached to each joint. The recorded displacement response was used to obtain the reference capacity curve maximum displacement that was used to evaluate the accuracy of the maximum displacement obtained from acceleration and the maximum displacement from SDOF analysis.

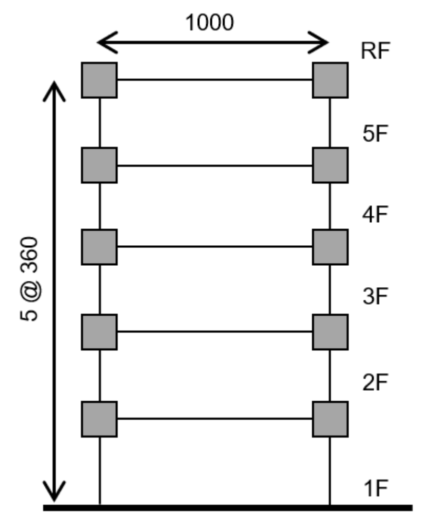


Figure 3. Layout of steel frame specimen

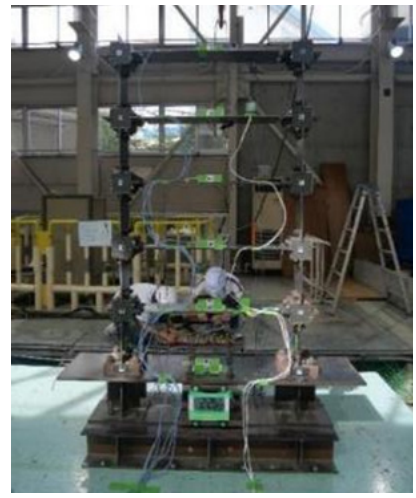


Figure 4. Photo of the steel frame specimen

Three specimens were constructed for the configuration shown in Figure 3, and each was subjected to a different seismic excitation, including: the 1999 Chi-Chi earthquake event; the 2011 Christchurch earthquake event; and an artificial earthquake record (WG60). For each seismic record, the specimen was tested under a sequence of excitations scaled from 20% to 200% in steps of 20%, where 100% corresponds to the elastic design spectra used in Japanese engineering practice for soil class type 2. The tests were numbered from Test 1 to Test 10 (see Table 1). Nonlinear deformation was observed at Test 5 for the Chi-Chi earthquake record and the Christchurch earthquake record, and Test 4 for the WG60 record.

Table 1. Experimental program

Test #	Earthquake records		
	Chi-Chi	Christchurch	WG60
1	20%	20%	20%
2	40%	40%	40%
3	60%	60%	60%
4	80%	80%	80%
5	100%	100%	100%
6	120%	120%	120%
7	140%	140%	140%
8	160%	160%	160%
9	180%	180%	180%
10	200%	200%	-

4. SDOF ANALYIS OF THE PROPOSED CASE STUDY

This section presents a detailed example to estimate the equivalent SDOF system's parameters from acceleration response.

4.1. Damping ratio estimation

The damping ratio corresponding to the first mode of the structure was estimated from the acceleration response to 20% event by using the half-power bandwidth method [6]. The frequency response function (FRF) was calculated using Eq. (4), in which: $S_{R,R}$ and $S_{T,T}$ are power spectral density of acceleration response at roof floor and at table floor, respectively; $S_{T,R}$ is cross power spectral density of acceleration response at table floor and at roof floor. Estimated damping ratios for SDOF analysis of three specimens that were tested under Chi-Chi earthquake record, Christchurch earthquake record and the WG60 record were 1.12%, 0.81%, and 1.17%, respectively.

$$FRF = \sqrt{\frac{S_{R,R}.S_{T,R}}{S_{T,T}.\left|S_{T,R}\right|}} \qquad (4)$$

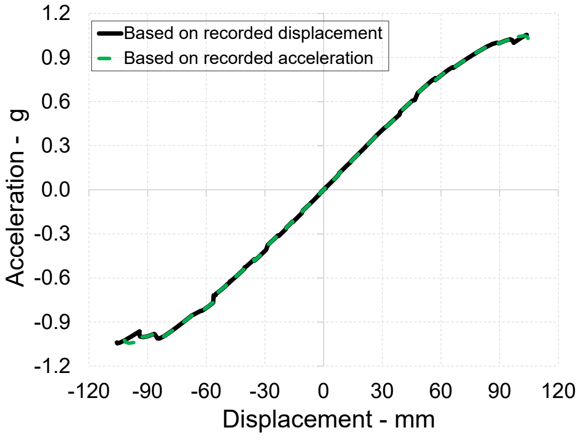
4.2. Bilinear capacity curve for SDOF analysis

Figure 5 compares the skeleton capacity curve obtained from acceleration and the reference curve obtained using recorded displacement response for Test 5 and Test 8 under Chi-Chi earthquake record. As can be seen, due to the lack of low-frequency displacement response, there was a significant error in the maximum displacement of the capacity curve obtained from acceleration data for the case with large nonlinear deformation. This phenomenon is similar for all other specimens.

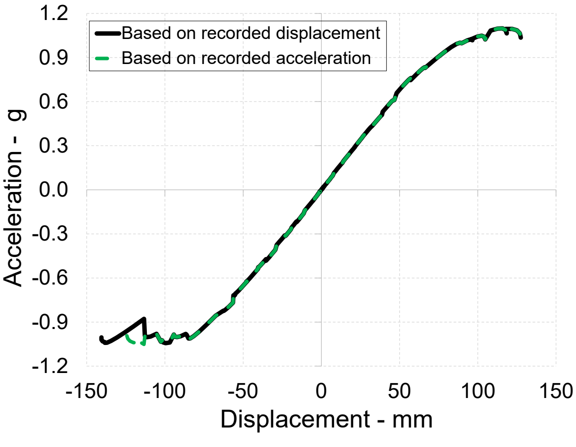
A bilinear capacity curve was fitted to the skeleton capacity curve estimated from acceleration data using an automatic procedure. The corner point of the bilinear curve was determined when the ratio between the equivalent load and the original load first exceeded . The best-fit bilinear capacity curve from the skeleton capacity curve was determined by firstly generating different bilinear curves with varied from 1.01 to 1.10 in steps of 0.01, in which the post-yield stiffness of the generated bilinear capacity curve must be not a negative value. Subsequently, the error of each generated bilinear capacity curve to the skeleton capacity curve was calculated using Eq. (5), which was proposed by Wang et al [7] (and are the equivalent load on the bilinear curve and the original load on the skeleton curve, respectively; *n* is the number of points on the skeleton capacity curve). Finally, the bilinear curve with the smallest error value and a positive post-yield stiffness was selected as a best-fit bilinear capacity curve. Figure 6 shows the fitted bilinear capacity curve from skeleton capacity curve of Test 8 under the Christchurch earthquake record.

$$\varepsilon = \sqrt{\frac{1}{N-1}\sum_{1}^{n}(A_1 - A_i)^2} \qquad (5)$$

An average bilinear curve of the capacity curve in positive and negative directions was derived from the estimated bilinear capacity curve in the positive and the negative directions. This involved the following steps: (1) calculate the average stiffness of each segment; (2) calculate average corner point displacement; and (3) calculate the corner point acceleration using average stiffness and displacement of the corner point. Figure 7 compares the average bilinear curve obtained after Test 8 under three earthquake records, respectively. There was a difference in the post-yielding stiffness between three bi-linear capacity curves which shows that the nonlinear behavior depends on the earthquake excitations. The curves, and the damping parameters identified previously, were then used for the SDOF analysis.



a) Test 5



b) Test 8

Figure 5. Skeleton capacity curves obtained using Kusunoki et al.'s method [2] – Chi-Chi earthquake record

4.3. Equivalent SDOF analysis

The SDOF analysis was performed with the bilinear hysteresis model and viscous damping model by using MATLAB R2022b software[8].

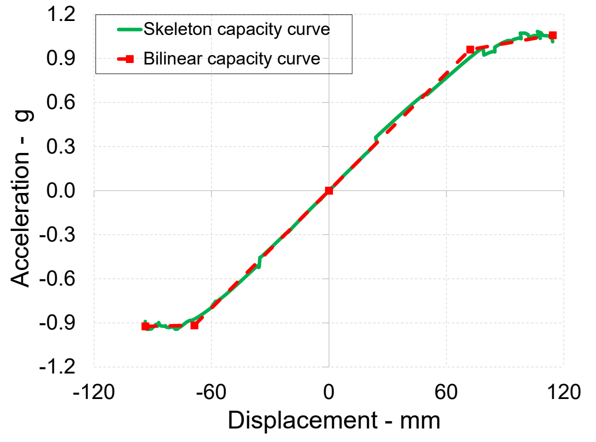


Figure 6. Bilinear capacity curves for 160% scale of the Christ-Church earthquake

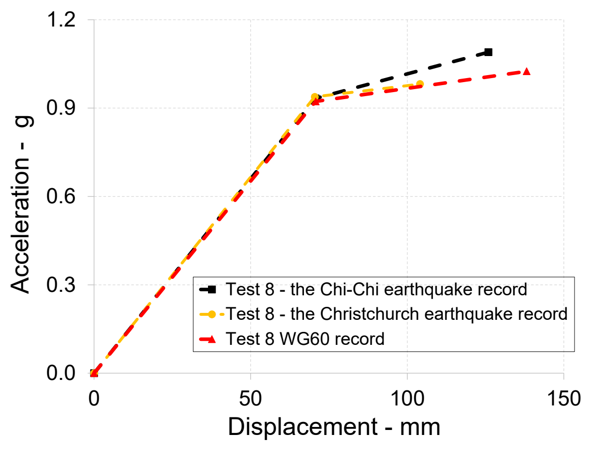


Figure 7. Average bilinear capacity curves for SDOF analysis of 160% events

5. RESULTS

Table 2 compares the maximum SDOF displacement to the maximum representative displacement obtained from floor acceleration response and the "Reference" obtained from floor displacement response. In the tables, AD is the maximum representative displacement obtained from floor acceleration response by using Kusunoki et al.'s [2] method; while SDOF is the maximum representative displacement obtained from SDOF analysis. The error was calculated as follows:

$$\text{error(mm)} = \left|x_{AD,SDOF}^{max}\right| - \left|x_{Reference}^{max}\right| \qquad (6)$$

In which $x_{AD,SDOF}^{max}$ is the estimated maximum representative displacement response from either applying Kusunoki et al.'s [2] method or from the SDOF analyses, and $x_{Reference}^{max}$ is the maximum representative displacement response based on recorded displacement.

As can be seen from Table 2, the maximum representative displacement error estimated from acceleration data using Kusunoki et al.'s [2] method was significant, with a maximum value of 55 mm for 200% event of Chi-Chi earthquake record. The SDOF maximum displacements were more comparable to the reference than the maximum displacement from acceleration. However, for some cases, the error was still significant with a maximum error of 31 mm for 200% event of the Chi-Chi earthquake. This could be due to the differences between bilinear hysteresis model and the real hysteresis response and/or the SDOF analysis did not consider the initial deflection of the specimen caused by previous events. The SDOF maximum displacement errors were only greater than the errors of maximum displacement from acceleration for cases with errors of maximum displacements from acceleration smaller than 5 mm and the difference was insignificant.

Table 2. Summary of results

Seismic records	Test #	Maximum displacement mm			Errors mm	
		Reference	AD	SDOF	AD	SDOF
Chi-Chi	5	106	105	109	-1	3
	6	116	119	121	3	5
	7	125	127	132	2	7
	8	141	127	144	-13	3
	9	168	135	151	-33	-17
	10	200	146	169	-55	-31
Christ church	5	95	93	90	-2	-5
	6	102	98	100	-4	-3
	7	111	108	115	-3	4
	8	122	114	120	-8	-3
	9	135	126	126	-9	-9
	10	144	131	135	-13	-9
WG60	4	100	98	108	-3	8
	5	124	113	116	-11	-8
	6	133	119	128	-14	-4
	7	152	127	142	-25	-10
	8	170	142	155	-28	-15
	9	192	146	173	-46	-19

Figure 8 compares the SDOF maximum displacements and the maximum representative displacement obtained from floor acceleration response to the reference maximum representative displacement. The solid line represents the estimated maximum representative displacement equal to the reference maximum representative displacement. Overall, the SDOF maximum representative displacement had higher accuracy than the maximum displacement from acceleration data. For cases with large nonlinear deformation, the error of SDOF maximum displacements were about half of the maximum representative displacement obtained from acceleration data.

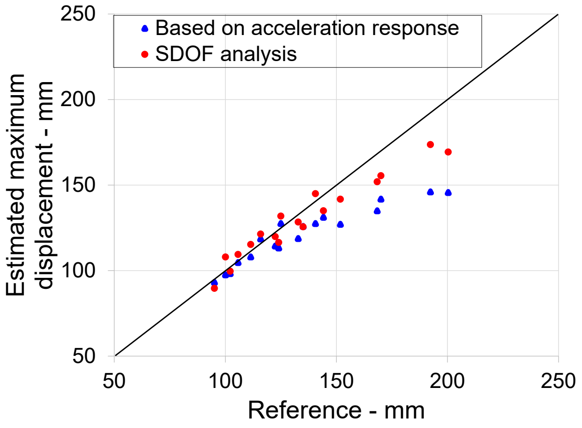
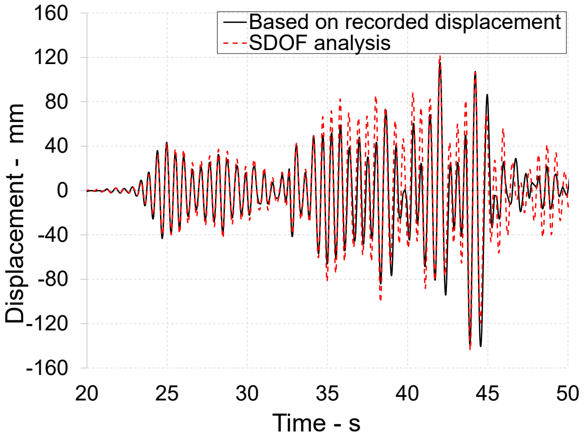
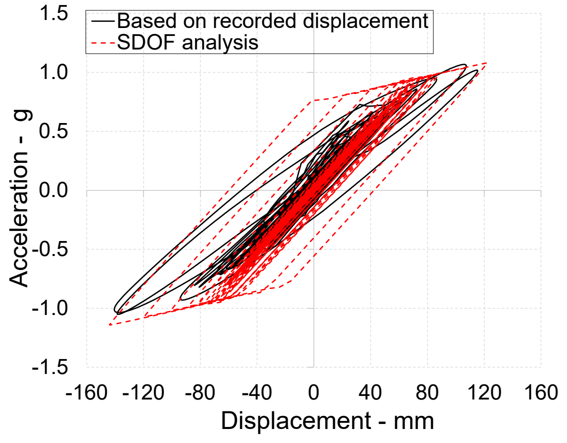


Figure 8. Comparison of estimated maximum representative displacements to the reference maximum representative displacement

Figures from 9 to 11 compare the SDOF analysis response to the reference response for Test 8 of three earthquake records, respectively. As can be seen, the SDOF displacement agreed well with the reference up to the maximum displacement, however there were slight differences in the hysteresis response. For the case of WG60 record, there was a difference in the direction of residual displacement. This may be due to the long duration of the earthquake wave and the presence of many high-magnitude acceleration peaks in both directions.

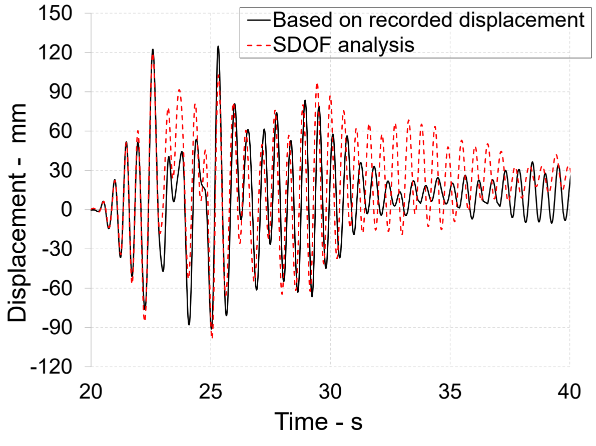


a) Displacement response

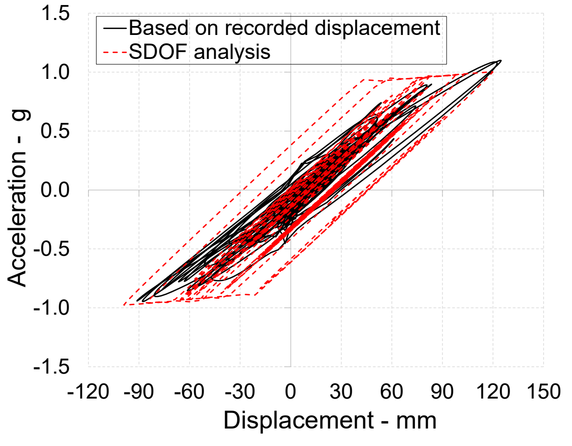


b) Hysteresis response

Figure 9. SDOF response of Test 8 – Chi-Chi earthquake record

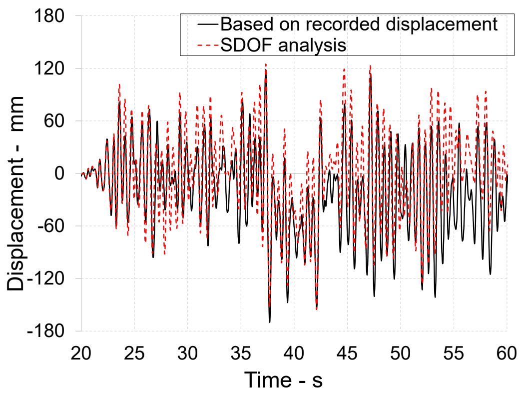


a) Displacement response

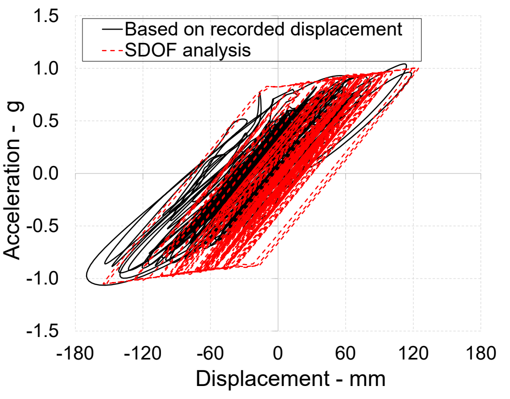


b) Hysteresis response

Figure 10. SDOF response of Test 8 – Christ-Church earthquake record



a) Displacement response



b) Hysteresis response

Figure 11. SDOF response of Test 8 – WG60 record

6. CONCLUSIONS

This study investigates the accuracy of equivalent SDOF analysis to estimate the maximum displacement response of buildings using parameters estimated from recorded floor acceleration data. The method was verified using experimental data from shaking table tests of a five-story steel frame structure subjected to three different seismic records. It was found that:

- Overall, the SDOF analysis provided a more accurate maximum displacement than that obtained by using Kusunoki et al.'s method [2].

- To reduce the degree of underestimation of building damage levels due to inaccuracies in the maximum representative displacement estimated from acceleration using Kusunoki et al.'s method [2], the SDOF analysis maximum displacement can be used.

- While equivalent SDOF analyses can be used to evaluate the response of a building under a future seismic event, inclusion of the residual displacement caused by previous seismic events needs to be taken into account.

- In the future, it is necessary to perform further verification of the equivalent SDOF analysis with various types of existing steel frame structures under different seismic characteristics.

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